STABILITY

Two curves are usually plotted on a Skew-T for each "sounding." One represents air temperature (T), the other the dewpoint temperature (T_d). From such a sounding plotting, many other key quantities may be evaluated.

It is important to pay attention to units. Unless stated otherwise, all units are the following: pressure and vapor pressure are in mb, T and T_d are in °K, and mixing ratio is in g/kg.

Vapor pressure e

Vapor pressure (e) is the partial pressure which water vapor contributes to the total atmospheric pressure. When the air is unsaturated, more water vapor molecules are evaporating than condensing, and e quantifies the pressure these molecules are exerting. However, under some circumstances the rate of evaporation equals the rate of condensation, and the atmosphere is said to be saturated. Vapor pressure at equilibrium (saturation) is called *saturation vapor pressure* e_s, and represents an upper limit for which vapor can remain gaseous. e_s, which is a function of T only, increases exponentially with T because warmer water vapor molecules have more kinetic energy and can break their bonds more easily to escape to the gaseous state. Theoretically, e_s may be calculated (in mb) from the Clausius-Clapeyron equation:

$$e_s = 6.11 \exp\left[\frac{L}{R_v}\left(\frac{1}{273.15} - \frac{1}{T}\right)\right] \qquad 1$$

where $R_v = 461.5$ J kg^{-1} K^{-1} is the water vapor gas constant, and $L = 2.5 \times 10^6$ J kg^{-1} is the latent heat of vaporization. Although Eq. (1) is quite suitable for most calculations, an empirical equation based on measurements of $e_s(T)$ exists which actually is more accurate, and is given by Wexler:

$$\ln e_s = \sum_{i=0}^{6} g_i T^{i-2} + g_7 \ln T \qquad 2$$

where $g_0 = -2.9912729 \times 10^3$, $g_1 = -6.0170128 \times 10^3$, $g_2 = 1.887643854 \times 10^1$, $g_3 = -2.8354721 \times 10^{-2}$, $g_4 = 1.7838301 \times 10^{-5}$, $g_5 = -8.4150417 \times 10^{-10}$, $g_6 = 4.4412543 \times 10^{-13}$, and $g_7 = 2.858487$. However, since Eq. (2) is impractical for ordinary computations, Bolton (1980) derived the following empirical equation based on Wexler's equation:

$$e_s = 6.11 \exp\left[\frac{17.67(T - 273.15)}{T - 29.65}\right] \qquad 3$$

Djuric (1994) offers this less accurate but easier to use formulation:

$$e_s = 6.11 \exp\left[\frac{19.8(T - 273.15)}{T}\right] \approx 6.11 \exp[0.073(T - 273.15)] \qquad 4$$

These vapor pressure relationships are useful for many meteorological applications, and for stability calculations on computers. However, these vapor pressure calculations are already "built-in" on the Skew-Ts in the mixing ratio plots (see next section).

Mixing ratio q

Mixing ratio (q) is the ratio of the mass of water vapor (m_v) present to the mass of dry air (m_d). In other words, $q = m_v/m_d$. Its units are expressed in parts per thousand, usually grams of water vapor per kilograms (g/kg) of dry air. (Often the symbol w is used, but I find this confusing since w also is used for vertical motion. Sometimes the symbol r is also used in meteorology literature).

For a given value of e and pressure p, q can be computed as:

$$q = \frac{0.622e}{p - e} \times 1000 \qquad 5$$

Note that the units of q are "kg per kg," which actually results in a mixing ratio value on the order of 10^{-3}. To convert to g/kg, we have multiplied by 1000. (Note that, since $e << p$, q is often calculated as $q \approx 0.622e/p \times 1000$. Therefore, e may be computed as a function of p and q. However, this can give unacceptable errors, particularly in warm, moist air masses, and should be avoided.)

To find q on a Skew-T at a given pressure, find the intersection of the mixing-ratio line that crosses T_d at p. On the sounding shown in Fig. 1, for example, T_d at 700 mb is $-13°$C, and $q = 2.0$ g/kg at that level.

Similarly, *saturation mixing ratio* (q_s) is defined as the mixing ratio the air would have if saturated. Obviously, q_s is a function of e_s (and hence temperature) and p, and given by

$$q_s = \frac{0.622e_s}{p - e_s} \times 1000 \qquad 6$$

To find q_s on a Skew-T, find the intersection of the mixing-ratio line that crosses T at p. On the sounding shown in Fig. 1, for example, T at 700 mb is $-5°$C, and $q_s = 3.8$ g/kg at that level.

Dewpoint temperature T_d

T_d is defined as the temperature to which air must be cooled isobarically (constant pressure) and constant mixing ratio to be saturated. If one knows p and q, T_d is found at the intersection point on a Skew-T.

It is frequently useful to calculate T_d from q. Since e_s is a function of T, it follows that e is a function of T_d. Therefore, by inverting Eq. (4) and solving for T_d gives:

$$T_d = 13.7 \ln \frac{e}{6.11} + 273.15 \qquad 7$$

or, alternatively, by inverting Eq. (3) one obtains

$$T_d = \frac{4880.23 - 29.65 \ln e}{19.48 - \ln e} \qquad 8$$

To compute T_d from q, one may use the approximation:

$$q \approx \frac{0.622e}{p} \times 1000$$

since $p >> e$ (again, only use this when precision is not important). By solving for e and substituting into Eq. (7) gives:

$$T_d = 13.7 \ln \frac{qp}{3800.42} + 273.15$$

Likewise, through substitution into Eq. (3) gives:

$$T_d = \frac{4880.23 - 29.65 \ln(qp/622)}{19.48 - \ln(qp/622)}$$

Specific humidity Λ

A quantity similar to mixing ratio is *specific humidity* (Λ). Specific humidity is the ratio of the mass of water vapor (m_v) present to the total mass of the air sample (m_d+m_v). In other words, $\Lambda = m_v/(m_d+m_v)$. Since $m_v < m_d$, generally $q \approx \Lambda$ for synoptic purposes. q is also easily measured, whereas Λ is not. However, using Λ is preferable for very precise theoretical work.

Relative humidity RH

Relative humidity (RH) is the ratio of the amount of water vapor in a volume of air to the amount that volume can "hold" if the air were saturated. It is usually multiplied by 100 and given in %. RH may be computed by:

$$\text{RH} = \frac{e}{e_s} \times 100 \qquad 9$$

or more commonly approximated as

$$\text{RH} \approx \frac{q}{q_s} \times 100 \qquad 10$$

From Fig. 1, RH at 700 mb is RH = 100(2.0/3.8) = 53%. RH is a function of q, T, and p. It is *not* an accurate measurement of moisture in the air for this reason (ever notice how much RH fluctuates during the day?). It is useful for agricultural purposes, since it indicates evaporative potential of foilage. Otherwise, better indicators of moisture are q and T_d. In fact, a "rule of thumb" for human discomfort is when $T_d \geq 65°$F.

This is also an alternative way to compute e. If RH is known (by first knowing q and q_s), then if one also knows e_s, e may be computed from Eq. (9).

RH may also be approximated from the "dewpoint depression" $T - T_d$ according to Djuric (1994):

$$\mathrm{RH} = \exp[-0.073(T - T_d)] \times 100 \qquad 11$$

Obviously, when RH = 100%, $T = T_d$, but RH is *not* T/T_d. However, balloon measurements of T and T_d rarely are the same due to the limited sensitivity of the hygrometer in the radiosonde. Typically, clouds are present when the dewpoint depression is 5°C or less (RH > 70%). This is because even saturated air is not saturated *everywhere*, but in patches, therefore a RH of 70% or more aloft indicates saturated air parcels are nearby.

BUOYANCY and STABILITY

The atmosphere is said to be "unstable" when an ascending air parcel (saturated or unsaturated) is less dense than the surrounding environment. Such an air parcel is also called "buoyant." An ascending air parcel's temperature will decrease with height even under adiabatic conditions. The adiabatic assumption states that an air parcel is "insolated" from the environment such that no heat is added or removed from the air parcel, and no interaction with the environment is allowed. This is a direct result of the First Law of Thermodynamics:

$$d\dot{Q} = d\mu + d\chi \qquad 12$$

where $d\dot{Q}$ is a differential increment of heating rate of the air parcel, $d\mu$ is the differential change in internal energy of the air parcel, and $d\chi$ is the differential element of work done by the air parcel. Under the adiabatic assumption, $d\dot{Q} = 0$, and $d\mu = -d\chi$.

The basic definition of work is "force times distance." Crudely speaking, if an air parcel exerts a force on the surrounding air, it is doing work on the surrounding environment during this expansion process, and $d\chi > 0$. If the surrounding air exerts a force on the air parcel, the surrounding environment is doing work on the air parcel as it contracts, and $d\chi < 0$. Pgs 61-63 in Wallace and Hobbs describe the details.

Internal energy consists of the mean kinetic energy of the air parcel's molecules (which controls temperature), and of forces of attractions between the molecules. However, experiments performed by J. P. Joule in the 1800's showed that attractive and repulsive forces between atmospheric molecules are not important, and therefore internal energy depends only on temperature (called Joule's Law, see pgs 61-64 in Wallace and Hobbs). This means that under an adiabatic transformation, if an air parcel exerts work on the environment, its internal energy (and temperature) must decrease. Likewise, for an adiabatic process, if an air parcel has work performed on it by the environment, its internal energy (and temperature) must increase.

The temperature decreases in an ascending air parcel because it is expanding as the pressure decreases, and when a gas expands in an environment which is not a vacuum (such as the atmosphere), some of the internal energy is being used to do work against the environment. Since the internal energy decreases, the temperature decreases. In essence, the molecules inside the air parcel "push" the sides of the parcel outwards as environmental pressure decreases, spending part of their kinetic energy in the process and thus slowing down. Therefore, since temperature is a measure of average kinetic energy, the parcel's temperature decreases.

The rate of temperature decrease an unsaturated air parcel will experience as it ascends can be computed from one form of the First Law (see pgs 64-65 in Wallace and Hobbs for details):

$$d\dot{Q} = c_p dT - \rho^{-1} dp \qquad 13$$

where $c_p = 1004$ J deg^{-1} kg^{-1} is the specific heat at constant pressure. By inserting the hydrostatic equation $dp = -\rho g dz$:

$$d\dot{Q} = c_p dT + g dz \qquad 14$$

and for an adiabatic process where $d\dot{Q} = 0$, Eq (14) may be solved to compute the rate of temperature change for an unsaturated ("dry") air parcel undergoing ascent or descent $[(dT/dz)_d]$ as:

$$\Gamma_d = -\left(\frac{dT}{dz}\right)_d = \frac{g}{c_p} \qquad 15$$

where Γ_d is the *dry adiabatic lapse rate*, and is defined to be a positive quantity. The value of Γ_d is 9.8 deg km^{-1}. This says that an unsaturated air parcel will experience a temperature decrease (increase) of about 10°C per 1 km ascent (descent). This is often referred to as "dry adiabatic" ascent or descent.

Eventually an air parcel ascends to some pressure level where condensate forms. It will then ascend as a "saturated" air parcel, releasing latent heat. This latent heat partially offsets the temperature decrease associated with adiabatic expansion. This is often referred to as "moist adiabatic" ascent or descent.

This latent heat release may be quantified as $d\dot{Q} = -L dq_s$, where L is the latent heat of condensation. Upon substitution into Eq (14):

$$\frac{dT}{dz} = -\frac{L}{c_p}\frac{dq_s}{dz} - \frac{g}{c_p} \qquad 16$$

and using the fact that:

$$\frac{dq_s}{dz} = \frac{dq_s}{dT}\frac{dT}{dz} \qquad 17$$

upon insertion of Eq (17) into (16), one can to compute the rate of temperature change for a saturated air parcel undergoing ascent or descent $[(dT/dz)_s]$ as:

$$\Gamma_s = -\left(\frac{dT}{dz}\right)_s = \frac{\Gamma_d}{1+(L/c_p)(dq_s/dT)} \qquad 18$$

The magnitude of Γ_s is *not constant*, unlike Γ_d, but depends on p and T. Furthermore, since $dq_s/dT > 0$ always, $\Gamma_s < \Gamma_d$. Typical values of Γ_s vary from 4°C km^{-1} at the surface in the tropics to typical values of 6-7°C km^{-1} in the middle troposphere. Note that, since q_s is small near the tropopause, $\Gamma_s \approx \Gamma_d$ in the upper troposphere. Also note on a Skew-T that the slope of Γ_s *decreases* for warmer temperatures, indicating that ascending saturated air parcels in warm, moist air masses have more buoyancy.

As a first approximation, one may roughly assess the atmosphere's stability by comparing the slope of the atmosphere's lapse rate $\gamma = dT/dz$ against the slope of nearby Γ_d and Γ_s lines on a Skew-T. γ is the "sounding" a radiosonde measures, and varies much on a daily (and hourly!) basis. γ varies daily to due synoptic influences such as temperature and moisture advection, as well as large-scale ascent and descent. For example, low-level warm air advection ahead of a cold front will steepen the lapse rate, while subsidence downstream of a ridge will lessen the lapse rate (and may even create temperature inversions where $\gamma < 0$). Large-scale ascent (by an approaching mid-level trough) also tends to steepen the lapse rate. Daily variations of γ occur due to solar radiation, cloud cover, etc. (Soundings in Jackson are taken at 6AM and 6PM LST — why is this a problem?)

It is clear from the discussion so far that Γ_d and Γ_s through a fixed point on γ at level p delineate a crude assessment of stability at that level. The steeper the lapse rate, the more unstable is the atmosphere, which is further complicated by what level p saturation might occur. The following rules apply for "eyeball" assessments of stability on a Skew-T (see Fig. 2):

Absolutely stable ($\gamma < \Gamma_s$)

Regardless of whether saturation occurs, the parcel will return to its original position if displaced from level p. This is because the parcel is more dense than the surrounding environment (negatively buoyant). Note that a temperature inversion (where the environmental T increases with height, or $\gamma < 0$), is always absolutely stable. This is the reason inversions can cause air pollution problems: polluted air parcels cannot escape from the lower layers because as they ascend they become more dense than the surrounding environment. The main types of inversions are: 1) frontal inversion; 2) subsidence inversion; 3) radiation inversion; and 4) turbulent inversion (see pg 79 in Djuric for details).

Absolutely unstable ($\gamma > \Gamma_d$)

The parcel will accelerate from its original position if displaced from level p. This is because the parcel is less dense than the surrounding environment (positively buoyant). This is not observed in soundings in reality because under conditions of absolute instability the atmosphere instantly overturns and become stable.

Conditionally stable ($\Gamma_s < \gamma < \Gamma_d$)

The parcel will accelerate away from its original position if displaced from level p and if saturated. On the other hand, if unsaturated the parcel will move back to its original position if displaced from level p. Such a stability state is common in the south and in the tropics. In other words, under such conditions an unsaturated ascending air parcel will be cooler than the surrounding environment, but should it be saturated and receive extra buoyancy from latent heat release, it will be unstable.

MORE PRECISE BUOYANCY CALCULATIONS

To determine stability in a more precise manner, the basic steps for a potentially ascending air parcel are: 1) raise an unsaturated air parcel "dry adiabatically" to where it will become saturated; 2) raise the saturated air parcel "moist adiabatically"; 3) determine if the parcel's air temperature is warmer (cooler) than the sounding's temperature. Where the air parcel is warmer, it is unstable; where it is cooler, it is stable. For an unstable situation, the ascending air parcel will accelerate upwards, resulting in cumulus clouds. For a stable situation, the ascending air parcel will sink to its original pressure level. In a situation, where a stable air parcel is "forced" to ascend (for example, by a front), stratus clouds result. We will discuss how to do this after a few cautionary on the accuracy of buoyancy calculations.

CAUTIONARY NOTE ON BUOYANCY CALCULATIONS

Determining stability is a tricky business. This is because the true temperature of an air parcel may be different from its dry/moist theoretical adiabatic value for a variety of reasons: 1) determining it's origin of ascent and original values often is unclear; 2) an ascending air parcel will "mix" with its surroundings laterally and vertically, which violates the adiabatic assumption. This mixing of an air parcel and the environment (called entrainment) will always decrease the air parcel's temperature; 3) the "pseudoadiabatic assumption" states that all condensation immediately falls out of a saturated air parcel. This often is not the case, and an ascending air parcel will often carry some condensate upwards with it. This "water-loading" increases it's density, which decreases its buoyancy; 4) drag of falling precipitation on upward vertical motion; 5) cooling from evaporation of falling precipitation; 6) interaction between the rising thermal and surrounding winds, especially if there is strong wind shear; and 7) the effects of ice formation (which releases latent heat) can sometimes be very important, and will enhance buoyancy.

We will not worry about these effects in our calculations, but be aware that they can be very important, dramatically changing what the air parcel's density will be compared to its theoretical value. Usually this results in less buoyancy than the theoretical value, but sometimes ice formation will give an unexpected buoyancy boost.

ASSESSING BUOYANCY ON A SKEW-T

Lifting Condensation Level LCL

The Lifting Condensation Level (LCL) is the height at which an unsaturated air parcel becomes saturated when it is lifted dry adiabatically from some initial height p with initial temperature T and initial dewpoint T_d. It is the level at which this dry adiabat intersects the q line associated with the initial T_d. This procedure is shown in Fig. 3.

Physically, the following situation is occurring: as an air parcel ascends, q_s will decrease as p and T decrease. However, q remains the same. When q_s decreases to the same value as q, saturation occurs (RH = 100%), and a cloud starts to form.

Even without a Skew-T, the LCL may be found. The temperature at the LCL (T_{LCL}) can be computed from the empirical relationship (Bolton 1980):

$$T_{LCL} = \frac{2840}{3.5 \ln T - \ln e - 4.805} + 55 \tag{19}$$

It will be shown in the next section that $T = constant \times p^{R/c_p}$ which is known as *Poisson's equation*. Therefore, the pressure level at the LCL (p_{LCL}) can be computed as:

$$p_{LCL} = p \left(\frac{T_{LCL}}{T} \right)^{c_p/R} \tag{20}$$

Typically the initial level from where the parcel is assumed lifted from is the surface, but the initial level may be at any pressure height. Sometimes, meteorologists assume some mean value in the boundary layer (about 100 mb thick) of T and q, and will compute the LCL from those mean values.

Level of Free Convection LFC

The LCL is not necessarily where the parcel becomes warmer than the surrounding environment. Often, even though the air parcel is saturated, it is still negatively buoyant. The Level of Free Convection (LFC) is defined as the height at which a parcel mechanically lifted will initially be less dense than the surrounding environment. The parcel will subsequently rise from its own buoyancy without needing any further mechanical lifting. The initial mechanical lifting which can force a negatively buoyant parcel to rise may be by a a front, low-level convergence, orography, or an upper-level trough.

To find the LFC, an air parcel is lifted dry adiabatically until saturated (at the LCL), then lifted moist adiabatically until it is warmer (less dense) than the surrounding air (see Fig. 4). The stable area bounded by the environmental sounding and the parcel's path is proportional to the amount of kinetic energy that must be supplied to move the parcel. This is called a *negative area* on a Skew-T. Likewise, the area where the parcel is buoyant is proportional to the amount of kinetic energy that the parcel gains from the environment, since it is accelerating upwards without any need for mechanical lifting. This is called a *positive area* on a Skew-T.

The level where a buoyant parcel's temperature again equals the environmental temperature is called the *Equilibrium Level* (EL). On Fig. 4, this is located at 590 mb. However, the parcel will not stop ascending at the EL. It will continue "overshooting," with it's vertical velocity decelerating since it is negatively buoyant. Severe thunderstorms may even penetrate 1-2 km into the stratosphere. Djuric illustrates a way to compute the overshooting top depth in Appendix L of his book.

CAPE

Ignoring the effects of virtual temperature, the acceleration of an air parcel by buoyancy is:

$$\frac{Dw}{Dt} = g\frac{T_{parcel} - T_{env}}{T_{env}} \tag{21}$$

The positive area on a Skew-T may be quantified by integrating the buoyant region from the LFC to EL. This quantity is called the *Convective Available Potential Energy* (CAPE). CAPE may be computed in z coordinates as:

$$\text{CAPE} = \int_{LFC}^{EL} g\left[\frac{T_{parcel} - T_{env}}{T_{env}}\right] dz \tag{22}$$

or, by inserting the hydrostatic equation and gas law, can be written in p coordinates as:

$$\text{CAPE} = -R\int_{p_{LFC}}^{p_{EL}} (T_{parcel} - T_{env})\, d\ln p \tag{23}$$

A crude method for calculating CAPE is shown in Appendix L of Djuric's book. Typically, CAPE values under 1000 m^2 s^{-2} indicate a small likelihood of strong convection, while values greater than 2000 m^2 s^{-2} show strong potential for severe weather.

An expression for computing the maximum vertical velocity at the EL (w_{max}) may now be derived based on CAPE. Note that Dw/Dt in Eq. (21) may be rewritten as:

$$\frac{Dw}{Dt} = \frac{Dw}{Dz}\frac{Dz}{Dt} \tag{24}$$

therefore, since $Dz/Dt = w$:

$$\frac{Dw}{Dt} = \frac{1}{2}\frac{D(w^2)}{Dz} \tag{25}$$

and by inserting Eq (25) into (21) gives:

$$\frac{\frac{1}{2}D(w^2)}{Dz} = g\frac{T_{parcel} - T_{env}}{T_{env}} \tag{26}$$

Now suppose we have an air parcel mechanically forced with initial vertical velocity w_o at the LFC. An expression for w_{max} may now be computed by multiplying Eq. (26) through by $2dz$ and integrating from the EL to LFC:

$$\int_{w_o}^{w_{max}} D(w^2) = 2\int_{LFC}^{EL} g\frac{T_{parcel} - T_{env}}{T_{env}} dz \tag{27}$$

Now note that the right hand side of Eq. (27) is just the definition of CAPE [see Eq (22)]. Therefore, the expression for w_{max} is:

$$w_{max} = [2\text{CAPE} + w_o^2]^{1/2} \qquad 28$$

Eq (28) will usually give overestimates of w_{max} since full buoyancy is rarely realized (see cautionary notes). Still, it is obvious that significant vertical motions can result from buoyancy if an air parcel can reach the LFC.

Whether a parcel can reach the LFC is determined by the amount of mechanical lifting, and how much negative area occurs on a Skew-T. Just like CAPE, an integration over the negative area from the initial pressure to the LFC may be performed. If this quantity, called *Convective Inhibition* (CIN), is too large, the low levels are too stable and thunderstorms will not develop.

CONSERVED QUANTITIES ON A SKEW-T

It is useful to define quantities which are *conserved* during dry and moist adiabatic motion. By conserved, it means the value remains the same regardless of it's location on a given adiabat. Such conserved quantities are useful in many meteorology applications, numerical models, and in creating thermodynamic diagrams such as the Skew-T.

Potential temperature θ

The *potential temperature* (θ) is defined as the temperature which the parcel would have were it expanded or compressed dry adiabatically from its existing p and T to a standard pressure level p_{ref} (usually taken as 1000 mb).

To find θ on a Skew-T, for any given p level, take the T at the level and bring it dry adiabatically to 1000 mb. The T value at 1000 mb is the potential temperature. For example, Fig. 5 shows how to find θ at 700 mb for the given sounding.

θ may also be computed without a Skew-T. By substituting the gas law into Eq. (13), and integrating from p_{ref} (where $T = \theta$) to p, we obtain:

$$\frac{c_p}{R}\int_\theta^T \frac{dT}{T} = \int_{p_{ref}}^p \frac{dp}{p} \qquad 29$$

which gives

$$\frac{c_p}{R}\ln\left(\frac{T}{\theta}\right) = \ln\left(\frac{p}{p_{ref}}\right) \qquad 30$$

and by taking the antilog of both sides:

$$\left(\frac{T}{\theta}\right)^{c_p/R} = \frac{p}{p_{ref}} \qquad 31$$

or

$$\theta = T\left(\frac{p_{ref}}{p}\right)^{R/c_p} \qquad 32$$

which is known as Poisson's equation. Often R/c_p is written as κ, which has a value of 0.286. Under dry adiabatic motion, θ remains constant.

Incidentally, it can be shown that $d\theta/dz > 0$ is equivalent to $\gamma < \Gamma_d$ (stable for unsaturated adiabatic transformations), and $d\theta/dz < 0$ is equivalent to absolute instability where $\gamma > \Gamma_d$.

Equivalent Potential Temperature θ_e

The *equivalent potential temperature* (θ_e) is the temperature a parcel would have if all its moisture were condensed out by a moist adiabatic process (with the latent heat release being used to heat the air parcel), and the parcel was then brought dry adiabatically back to 1000 mb. θ_e is conserved for both moist and dry adiabatic processes, so therefore moist adiabatic lines are also called θ_e lines.

The procedure for finding θ_e on a Skew-T is shown in Fig. 6. An air parcel is lifted from p to the LCL (Step 1), then lifted moist adiabatically to the top of the Skew-T where $q \approx 0$ (Step 2; note that the moist adiabat parallels the dry adiabats in this region). In Step 3, the air parcel is brought dry adiabatically back to 1000 mb, and the temperature is θ_e. Some charts are labeled with θ_e values at the top, thereby Step 3 is not needed. Note that large values of θ_e generally symbolize large buoyancy values.

A similar value, called *equivalent temperature* (T_e), is the temperature a parcel would have if all its moisture were condensed out by a moist adiabatic process (with the latent heat release being used to heat the air parcel), and the parcel was then brought dry adiabatically back to its original pressure. An example is shown in Fig. 6.

The derivation of the equation for θ_e is shown on pgs 78-79 in Wallace and Hobbs (1977), and will not be shown here. The resulting theoretical equation is:

$$\theta_e = \theta \exp\left(\frac{Lq_{LCL}}{1000c_pT_{LCL}}\right) \qquad 33$$

where q_{LCL} is the saturation mixing ratio at the LCL (note this is also q on the T_d profile from where the parcel is lifted). Likewise, the equation for T_e is:

$$T_e = T \exp\left(\frac{Lq_{LCL}}{1000c_pT_{LCL}}\right) \qquad 34$$

It turns out Eq. (33) works generally well in the mid-latitudes, but will underestimate θ_e in warm, moist air masses, and always underestimate θ_e in the tropics. This is because the specific heat of water at constant pressure (which is neglected in the derivation of Eq. (33)), is important under those conditions. This can result in θ_e errors of 4°K. A better equation is the empirical equation by Bolton (1980):

$$\theta_e = \theta \exp\left[q_{LCL}\left(1 + 0.81 \times 10^{-3} q_{LCL}\right)\left(\frac{3.376}{T_{LCL}} - 0.00254\right)\right] \qquad 35$$

It should be noted that the Skew-T ignores the specific heat of water, therefore Γ_s and θ_e are underestimated in the tropics.

For less accurate calculations, θ_e in Eq (33) may be approximated as:

$$\theta_e \approx \theta \left(1 + \frac{Lq_{LCL}}{1000 c_p T_{LCL}}\right) \qquad 36$$

since $(Lq_{LCL})/(1000c_pT_{LCL}) \approx 0.1$, the approximation $\exp(x) \approx 1 + x$ for small x may be used. Therefore, an estimate for typical atmospheric values from Eq (36) show that $\theta_e \approx 1.1\theta$ (Dutton 1976).

USING θ_e TO ITERATE FOR T IN AN ASCENDING SATURATED AIR PARCEL

Since θ_e is conserved for moist ascent, one may also compute the air parcel's temperature at any pressure level (assuming no entrainment of the air parcel) once θ_e is known. This is easily accomplished graphically on a Skew-T. However, should a Skew-T not be available, it is still possible to accomplish this using an iteration program.

This is accomplished by rewriting Eq. (33) as a function of temperature and pressure. By substituting Eq. (6) and Eq. (32) into Eq. (33), one obtains:

$$\theta_e = T\left(\frac{p_{ref}}{p}\right)^{R/c_p} \exp\left(\frac{0.622Le_s(T)}{c_pT(p - e_s(T))}\right)$$

Once θ_e is known, one may solve for T in an ascending saturated air parcel at any pressure level above the LCL and below the equilibrium level. This may be accomplished by rewriting the above equation as:

$$Res = \theta_e - T\left(\frac{p_{ref}}{p}\right)^{R/c_p} \exp\left(\frac{0.622Le_s(T)}{c_pT(p - e_s(T))}\right)$$

where Res is a *residual term*. By using an iteration technique (such as the *bisection method*), one progressively guesses different values of T until the iteration converges towards a solution where Res becomes small. Details about the bisection method are described in a separate handout.

OTHER SKEW-T PARAMETERS

Wet-bulb Temperature T_w

The *wet-bulb temperature* (T_w) is the lowest temperature air at constant pressure can be cooled by evaporating water into it. T_w is typically measured using a psychrometer. T_w is useful for estimating surface temperature due to evaporation from rainfall or a wet surface.

T_w may also be computed on a Skew-T. Figure 7 illustrates the method for finding T_w at a given pressure level (in this case, 700 mb). The procedure is to find the LCL, then follow a moist adiabat back to the original pressure. In this example, $T_w = -8$°C at 700 mb. T_w may also be computed using the bisection method by calculating θ_e and iterating for T at the surface pressure level.

Another quantity, the *wet-bulb potential temperature* (θ_w), is the wet-bulb temperature and air parcel would have if brought moist adiabatically to a reference pressure (usually 1000 mb). Figure 7 shows an example of computing θ_w.

Convective temperature T_c

The *convective temperature* (T_c) is the surface temperature that must be reached to start the formation of convective clouds by solar heating of the surface air. This is the surface temperature corresponding to a dry adiabatic lapse rate (created as a result of solar insolation and resultant mixing) which is warm enough so that parcel ascent from the shallow superadiabatic layer near the surface reaches a height at which condensation occurs.

To find T_c, the *Convective Condensation Level* (CCL) must be found first. The CCL is the height to which a parcel of air, if heated sufficiently from below, will rise adiabatically until it is just saturated. Often this corresponds to the height of the base of cumuliform clouds due to surface heating. Condensation at this height is manifested initially by shallow cumulus clouds which represent the tops of turbulent eddies within the boundary layer. The CCL is always higher than or equal to the LCL.

Once the CCL is attained, surface temperatures generally do not exceed T_c as a result of the shading of the ground by the clouds and the increased winds near the surface as the clouds themselves begin to enhance mixing within the layers below the CCL. T_c often closely corresponds to the maximum daytime surface temperature during the warm months. Likewise, the onset of cumulus cloud formation can be predicted by forecasting when the surface temperature will reach T_c. Such information is available from numerical models. But do remember that the general synoptic and local environment, as well as other instability mechanisms, will also control when convection will form. Nevertheless, the use of T_c as a forecast tool is most productive during the summer when synoptic forcing is weak and the forecast site is surrounded by a maritime tropical air mass. Hence, using T_c as a proxy for predicting air mass thunderstorms is quite effective. Generally speaking, air mass thunderstorms will occur under the following conditions: 1) if RH $> 40\%$ to 10,000 feet above cloud base; 2) if the EL is above the freezing point (0°C isotherm). However, thunder and lightning don't occur unless the cloud top is between -10°C and -20°C; 3) if wind shear is not too large, which would shear off the cloud tops.

Figure 8 shows an example for finding the CCL and T_c. To find the CCL, proceed upward along the mixing ratio line corresponding to the surface dewpoint temperature until this line intersects the T curve on the sounding. The CCL is at the height of this intersection. (Since there is much variation in q near the surface, sometimes an average q is calculated in the boundary layer or in the bottom 100 mb). To find T_c, from the CCL proceed downward along the dry adiabat to the surface-pressure isobar. The temperature read at this level is T_c (see Fig. 8).

Mixing Condensation Level MCL

The *Mixing Condensation Level* (MCL) is the lowest height that condensation will occur as a result of strong winds mixing a layer so as to attain uniform θ and q. The effect of surface heating is ignored to find the MCL. The MCL is located at the intersection of the mean q of the layer with the mean dry adiabat of the layer.

The determination of the MCL is shown in Fig. 9. It first requires an estimate or forecast of the height of the top of the layer to be mixed. There is no clear-cut procedure for this, and experience of a local area is the best teacher. However, a subjective estimate would consider boundary layer wind speeds, terrain roughness, and the location of a "turbulence inversion" on the sounding.

Once the top of the mixed layer is estimated, the procedure is as follows. Since θ and q of a completely mixed layer unsaturated layer are constant throughout the layer, an "equal-area" approximation of the mean θ and q may be used. This is done by finding equal areas of the mean dry adiabat (θ) and q with regards to the T and T_d lines as shown in Fig. 9. If an MCL exists, it will be within the mixed layer at the intersection of the mean q and mean θ lines. If there is no intersection of these two lines *within* the mixed layer, then the mixed air is too dry to reach saturation by the mixing process.

Density temperature T_ρ

Density temperature (T_ρ) is defined as the temperature at which dry air at the same pressure would have the same density as moist, cloudy air. T_ρ technically should be used in most thermodynamic calculations, including stability evaluations. This is because the water vapor dependence of density has been absorbed in the derivation of the ideal gas law, which allows meteorologists to use the dry gas constant (R) instead of the moist gas constant (R_v) and simplifies the thermodynamic equations. Therefore, T_ρ may be thought of as a "correction term" for temperature so as to account for the true density of the atmosphere when moisture and suspended liquid condensate are included.

The equation for T_ρ is:

$$T_\rho = T\frac{1 + q/622}{1 + .001q_{total}} \qquad 38$$

where q_{total} is the cumulative total of the mixing ratio q and any condensed, suspended liquid water content (q_l) such as in a cloud. (Recall that mixing ratio units are g/kg in these notes.) When no condensed water exists, Eq. (38) reduces to the *virtual temperature* T_v, which is defined as the temperature at which dry air at the same pressure would have the same density as moist air. The equation for T_v is:

$$T_v = T\frac{1 + q/622}{1 + .001q} \qquad 39$$

which may be simplified (see appendix at end of notes) as:

$$T_v \approx (1 + 0.000608q)T \qquad 40$$

An example of a T_v plot is shown in Fig. 10. Fortunately, the differences between T, T_v, and T_ρ are small, and in practical operations ignored. However, since $T_v > T$, T_v *adds* buoyancy to an air parcel. An increase of q by only 1 g/kg can increase CAPE by 20%, and w_{max} by 10%. On the other hand, suspended liquid condensate can reduce buoyancy as indicated in T_ρ [the "water-loading" effect, where it can be shown that $T_\rho \approx (1 + 0.000608q - 0.001q_l)T$]. As a rule of thumb, 3 g/kg of suspended liquid condensate decreases buoyancy by 1°C. It should be noted that the concept of "density temperature" is fairly new, and was first defined by Emanuel in 1994, but that the definition of T_v has spanned for decades.

Precipitable Water PW

Precipitable Water (PW) is defined as the vertical integral of water depth if all water vapor in a column of air were condensed out over a square meter of the earth's surface:

$$\mathrm{PW} = \int_0^\infty \rho\Lambda dz = \int_{p_{top}}^{p_{sfc}} \frac{\Lambda}{g} dp \qquad 41$$

where usually $q \approx \Lambda$ is assumed. PW shows moisture content better than stability parameters, and is often used to compliment stability calculations for convective situations. PW has units of kg m^{-2}, but it more commonly expressed in mm. Since the density of water is 1000 kg m^{-3}, 1 kg of water, when spread over the area of 1 m^2, makes a fallen precipitation layer of 1 mm. (Incidentally, 1 g cm^{-2} equals 1 cm of water depth.) PW can be used to estimate average precipitation potential over an area. PW values of 25 mm or more are generally sufficient to support showers and thunderstorms, but in elevated regions such as Colorado and Wyoming 10 mm is sufficient. PW is also used to assess flash flood potential.

Layer instability

Instability may occur not just by parcels being lifted, but by an atmospheric layer being lifted which makes that layer more vulnerable to parcel instability. Such a situation can occur when the bottom of a lifted layer becomes saturated but the top of the lifted layer remains unsaturated. This will increase the slope of the environmental lapse rate γ, and therefore increase the instability of that layer. Such a situation which makes the atmosphere more prone to instability by lifting of a layer is called *layer instability.*

Layer instability is more commonly called *convective instability* in meteorology literature, since convection results when the bottom of a lifted layer becomes saturated. However, convective instability is a poor and misleading term since it implies that convection does not result from the other instabilities where no lifting of a layer is required. Another common term for layer instability is *potential instability*, since there is the potential for convection if the bottom of a lifted layer saturates. In these notes, "layer instability" will be the preferred term, although further commentary on the synonym "potential instability" is attached to the end of this section.

The criterion for layer instability (stability) is when the *wet-bulb lapse rate is greater (less) than* Γ_s. In other words, one compares the slope of T_w to Γ_s. Figure 11 shows such a situation. However, T_w is usually not plotted on soundings, so another technique must be used to find layer instability. Before this other technique is described, let's discuss how lifting a stable layer can result in instability.

Figure 12 shows how a stable layer can become unstable when lifted. The unsaturated layer with lapse rate AB is absolutely stable in its original position between 700 and 800 mb. However, during the process of lifting this layer, the bottom part saturates while the top remains unsaturated. The new lapse rate A'B' is now steeper (conditionally unstable) and, since it is saturated, unstable.

Now, consider the fact that the lifting of the layer initially proceeds dry adiabatically (i.e., constant θ) from both A and B. However, when the bottom becomes saturated, the lower part of the layer proceeds moist adiabatically (i.e., constant θ_e) to A'. In the meantime, the top of the layer is still unsaturated for the same respective distance of lifting, and will not become saturated until at least B' (or even higher in this example).

As evident by this example, *another criterion for layer instability is that* $\partial\theta_e/\partial z < 0$. Likewise, it can also be shown that $\partial\theta_w/\partial z < 0$ *indicates layer instability*. Furthermore, when θ_e or θ_w increase with height in a layer, the depth of that atmosphere is stable to lifting.

As a qualitative guide, dry air above moist air is a fingerprint of layer instability, and is one criteria looked for in predicting severe thunderstorm outbreaks. Layer instability usually is indicated by a subsidence sounding as shown on pg 79 in Djuric. (Notice that, while an inversion is absolutely stable, lifting by a dry line or a front of a layer containing this inversion can lead to a convective outbreak!) Conversely, layer stability is characterized by relatively dry air capped by relatively moist air aloft. Layers of a sounding where no amount of lifting will result in instability are indicative of layer stability.

Finally, one has to be careful in the interpretation of layer instability, and there is much confusion in the literature about it. As noted by Emanuel (1994), layer instability is only valid in a saturated atmosphere. Even though θ_e may be computed at anytime, it really only has any physical meaning when the atmosphere or an air parcel becomes saturated. In fact, the atmosphere may be unstable, neutral, or stable regardless of whether θ_e increases or decreases with height. Furthermore, even if θ_e decreases with height, should the layer not be lifted, then no instability will occur (at least by the mechanism of layer instability).

The state of θ_e decreasing with height is, however, one of *potential instability*, as remarked earlier in the notes. The instability is potential in this sense: were the *entire air mass* lifted until it's bottom becomes saturated, then the upward decrease of θ_e does imply instability. Therefore, at some point in the lifting process, the air mass will become unstable in the parcel sense. An example of this is the flow of potentially unstable air into the ascent region of a mid-latitude cyclone. As the layer ascends, it becomes unstable and convection breaks out.

REFERENCES

Bolton, D., 1980: *The computation of equivalent potential temperature.* Mon. Wea. Rev., **108**, 1046-1053.

Djuric, D., 1994: *Weather Analysis.* Prentice Hall, 304 pp.

Dutton, J. A., 1976: *The Ceaseless Wind. An Introduction to the Theory of Atmospheric Motion.* McGraw-Hill, Inc., 579 pp.

Emanuel, K. A., 1994: *Atmospheric Convection.* Oxford University Press, 580 pp.

Wallace, J. M., and P. V. Hobbs, 1977: *Atmospheric Science. An Introductory Survey.* Academic Press Inc., 467 pp.

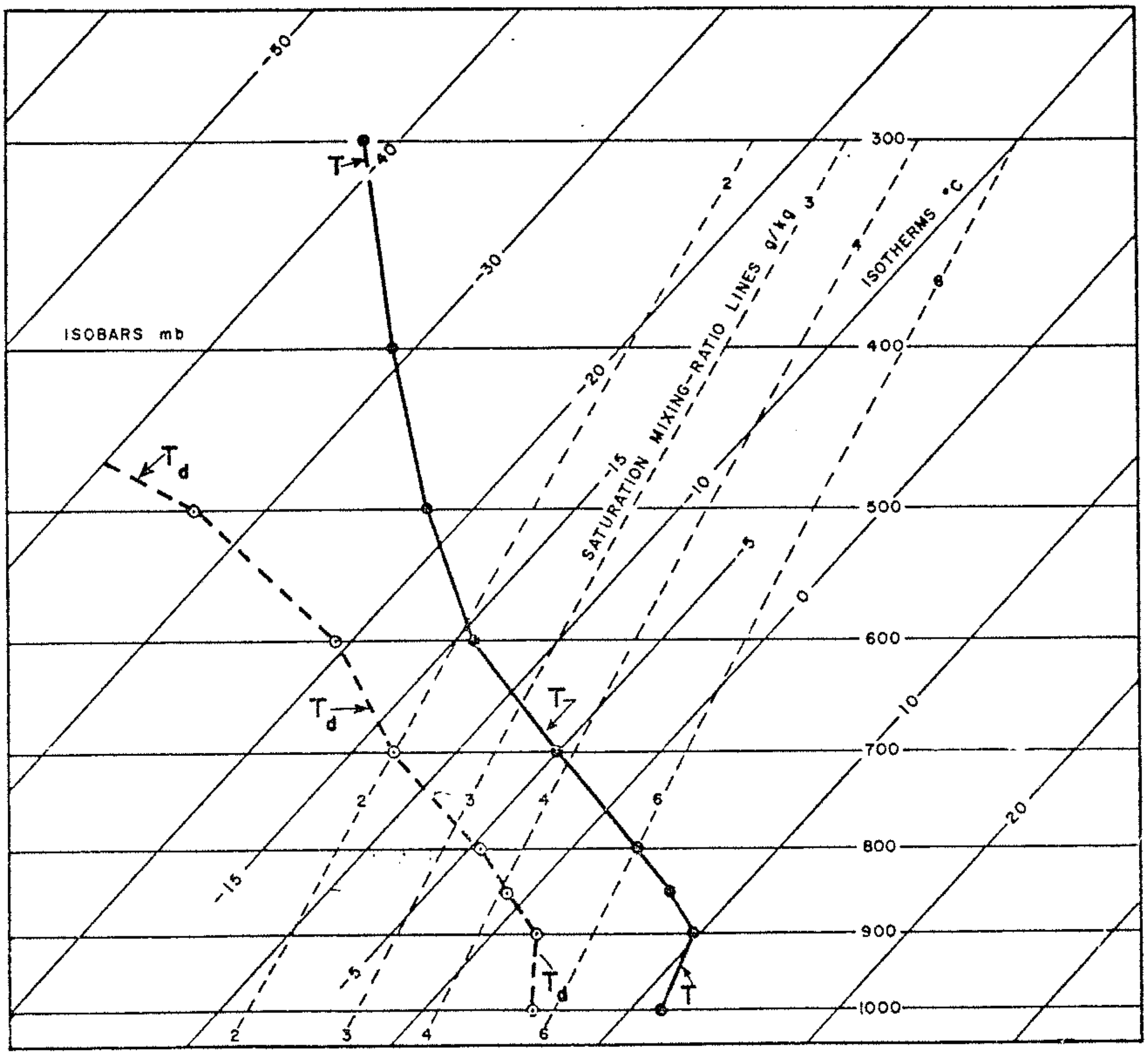

Figure 1 . Sample Sounding on the Skew-T Chart.

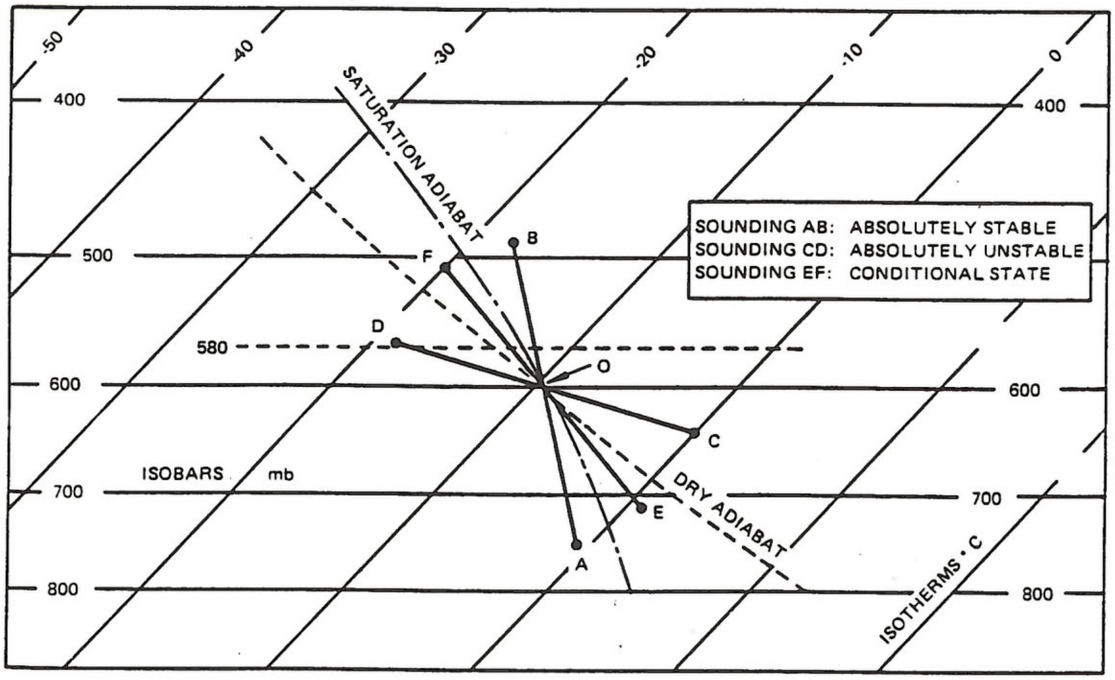

FIGURE 2 STABILITY CLASSIFICATIONS.

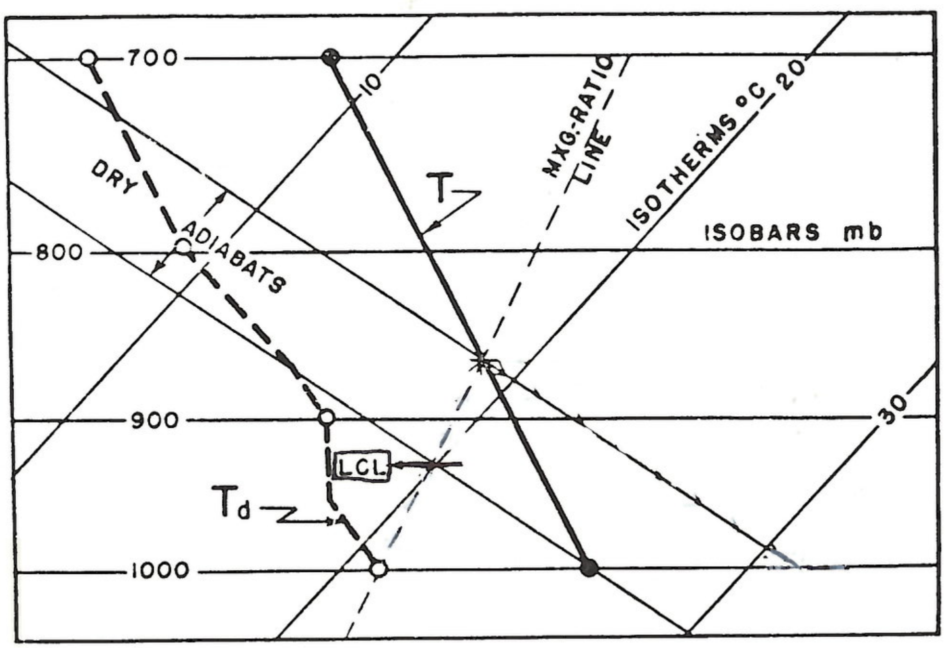

Figure 3. Lifting Condensation Level example

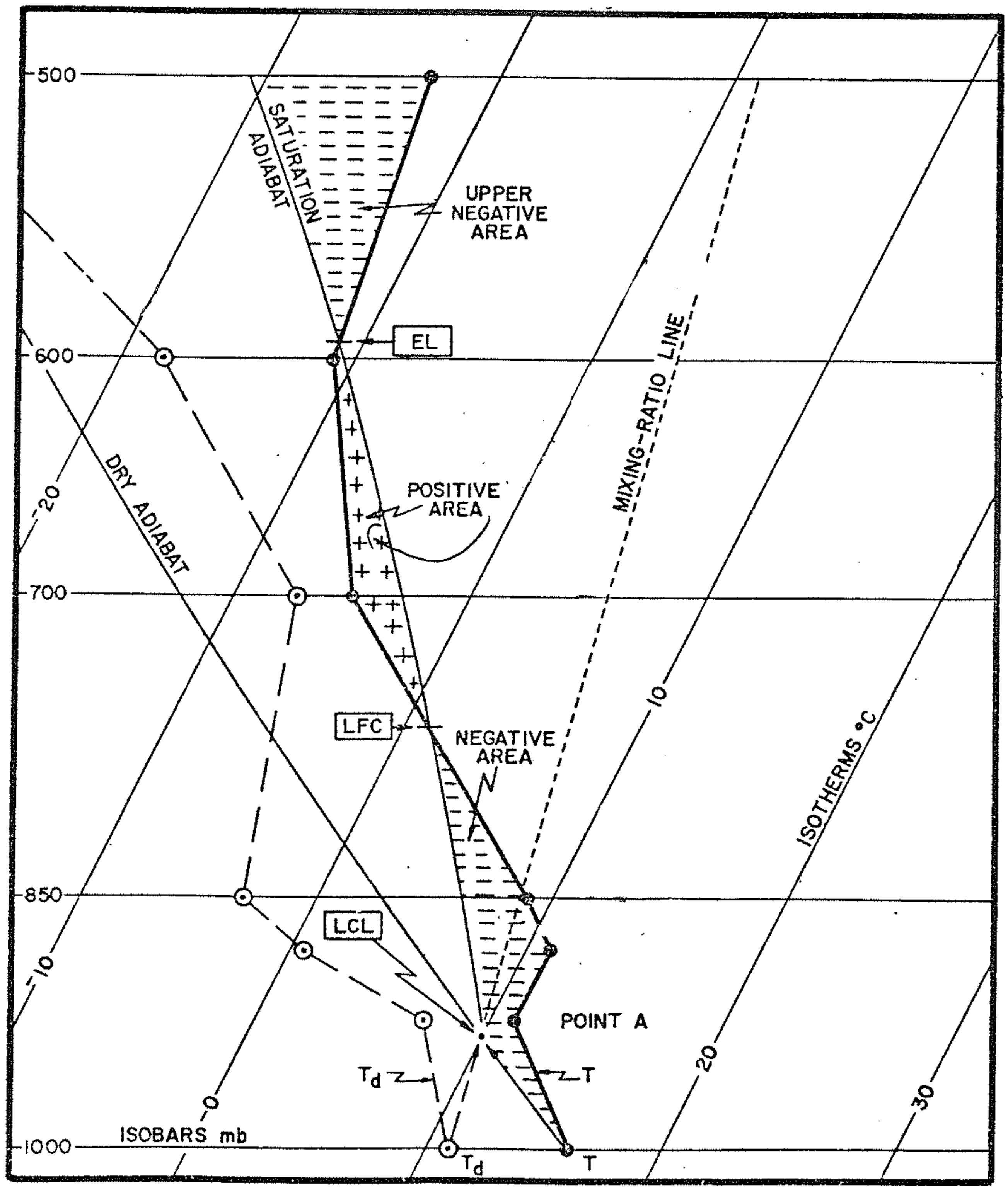

Figure 4 Determination of the Positive and Negative Areas on a Sounding Due to the Lifting of a Surface Parcel.

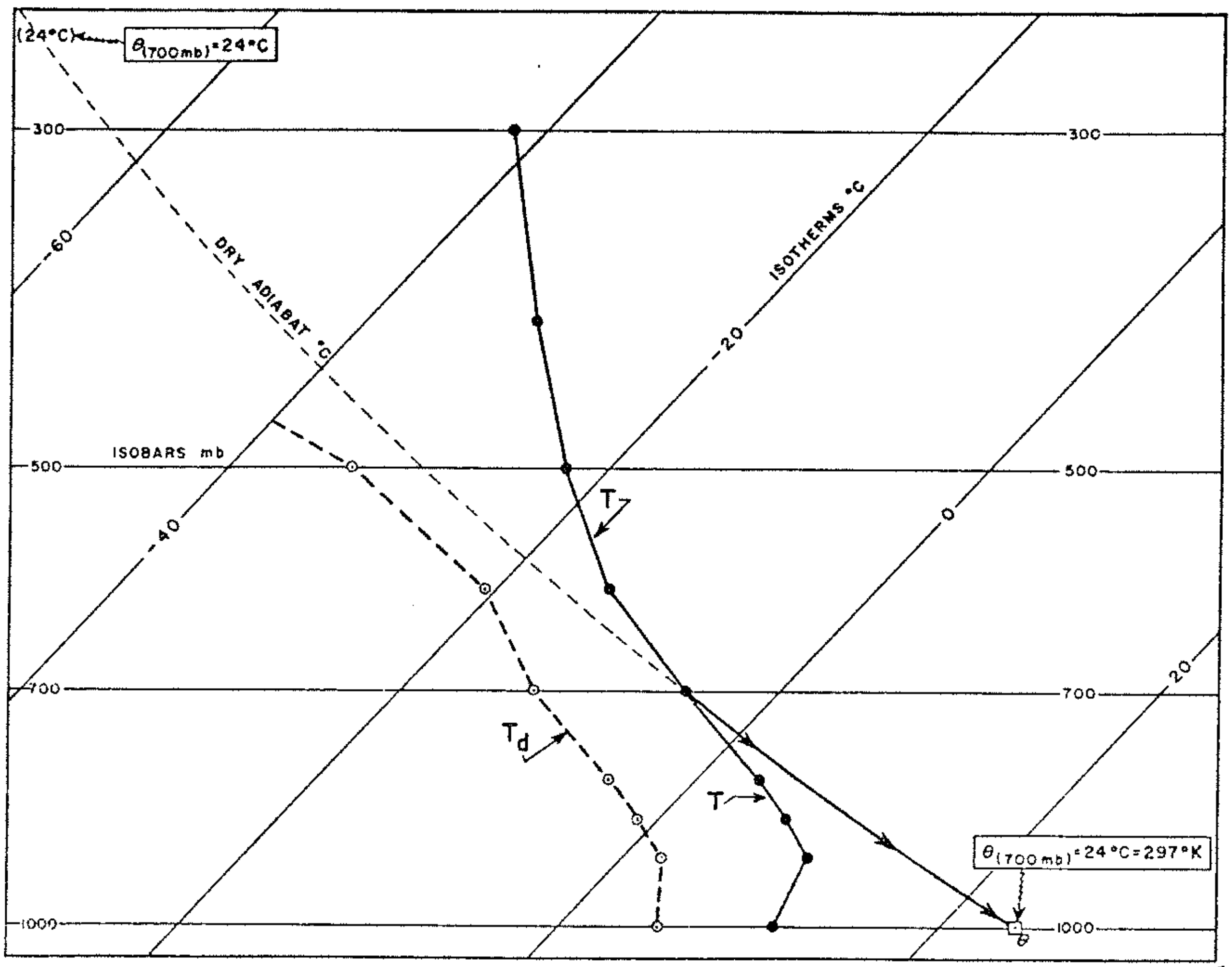

Figure 5 Determination of the Potential Temperature (θ).

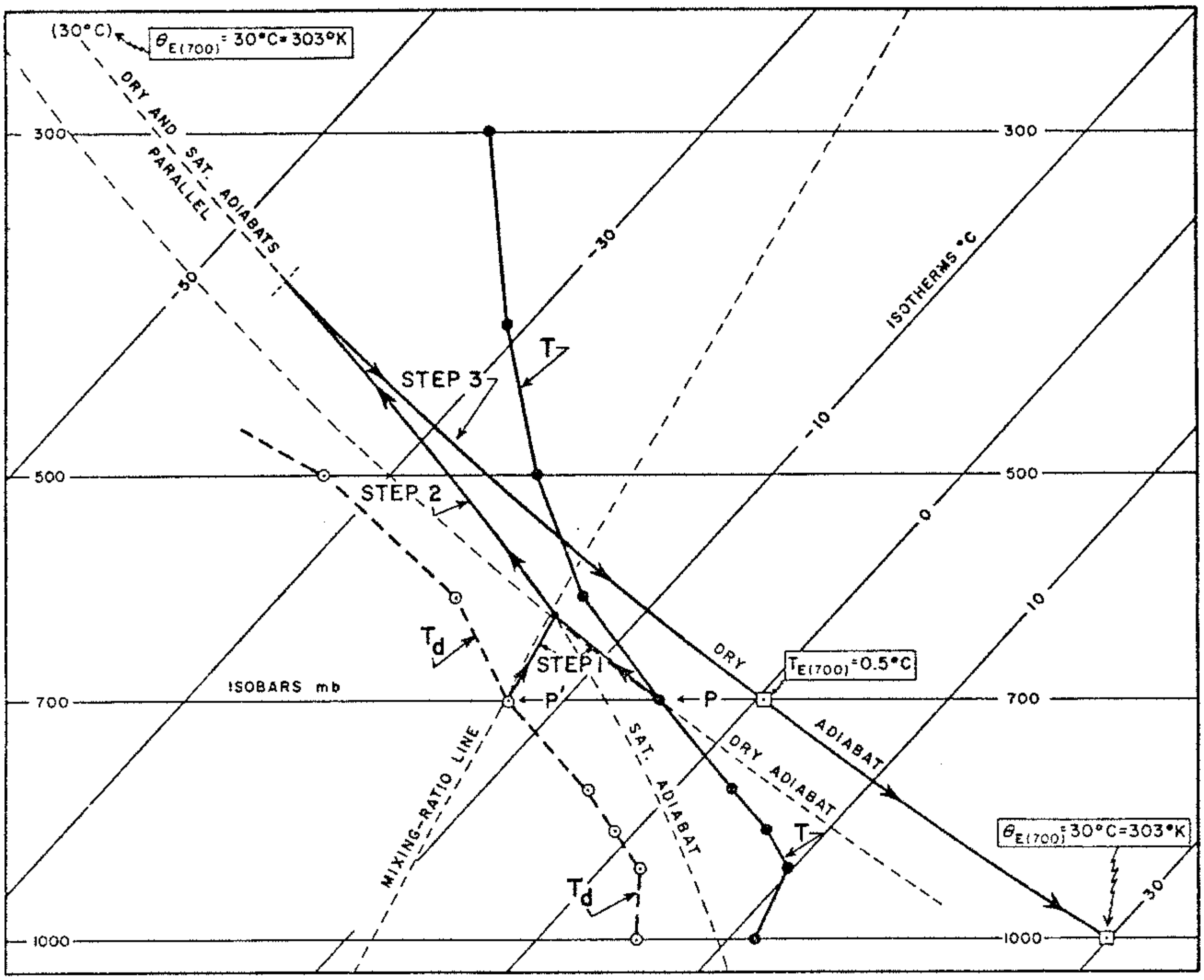

Figure 6 Determination of the Equivalent Temperature (T_E) and the Equivalent Potential Temperature (θ_E).

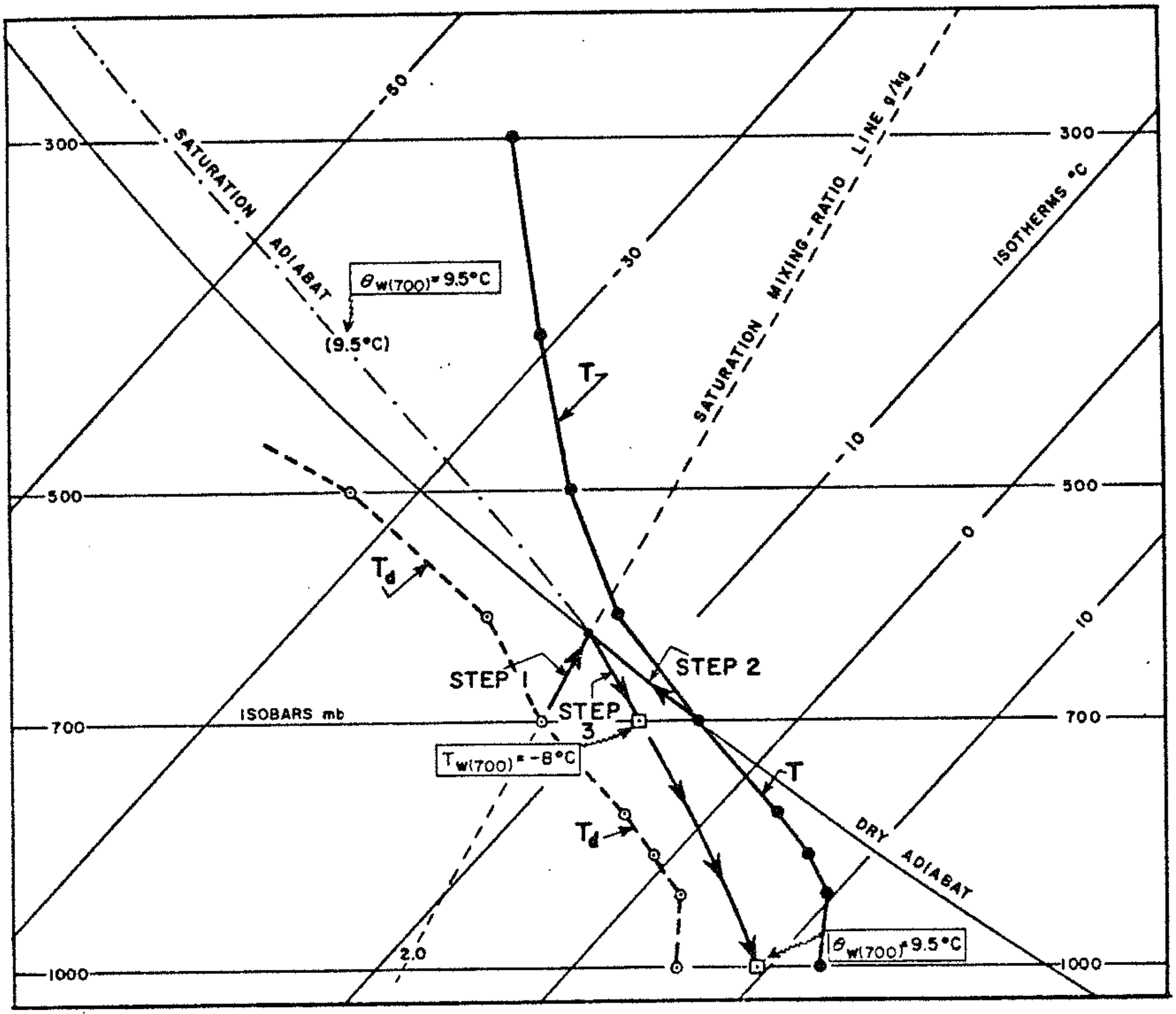

Figure 7. Determination of the Wet-Bulb Temperature (T_w) and the Wet-Bulb Potential Temperature (θ_w).

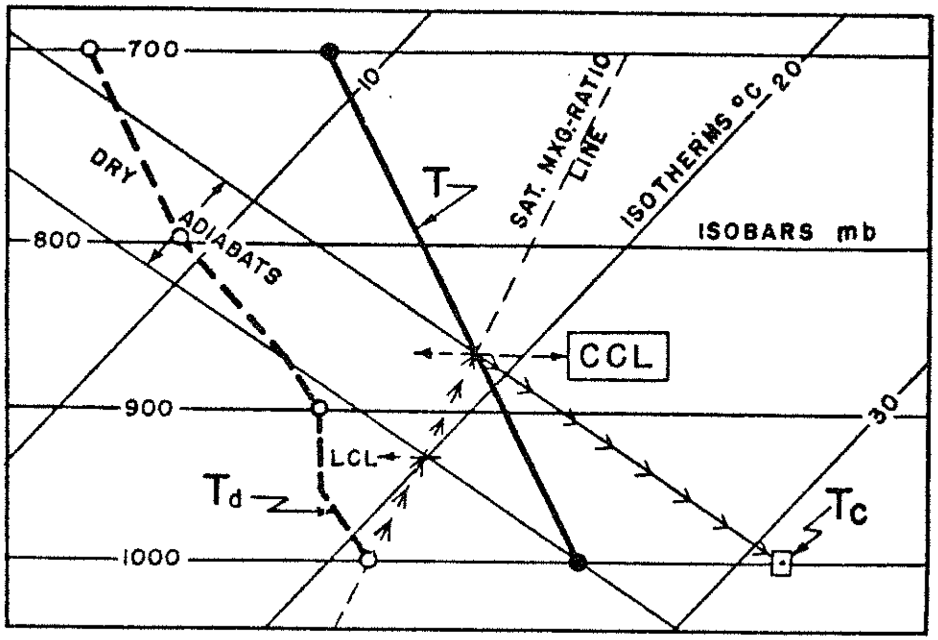

Figure 8 Procedure for Locating the Convection Condensation Level, the Convection Temperature, and the Lifting Condensation Level.

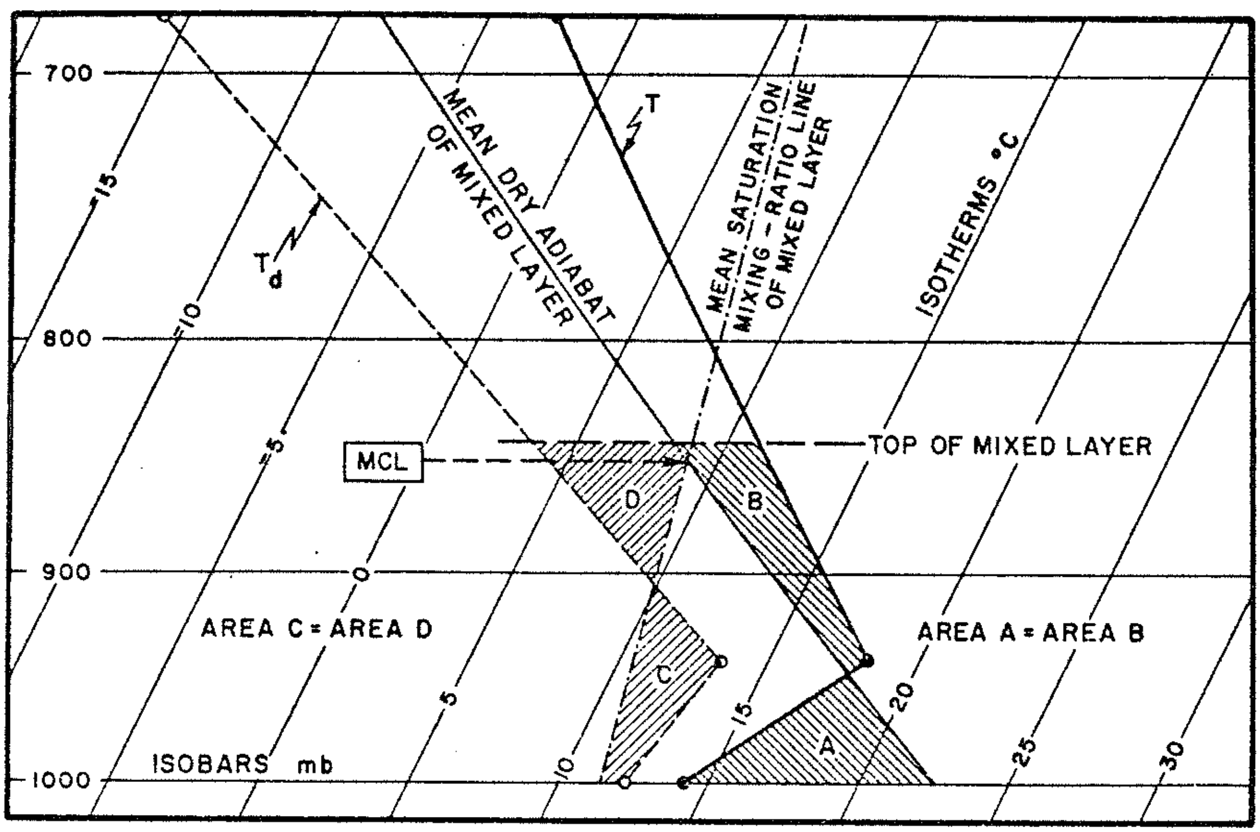

Figure 9 Determination of the Mixing Condensation Level on a Sounding.

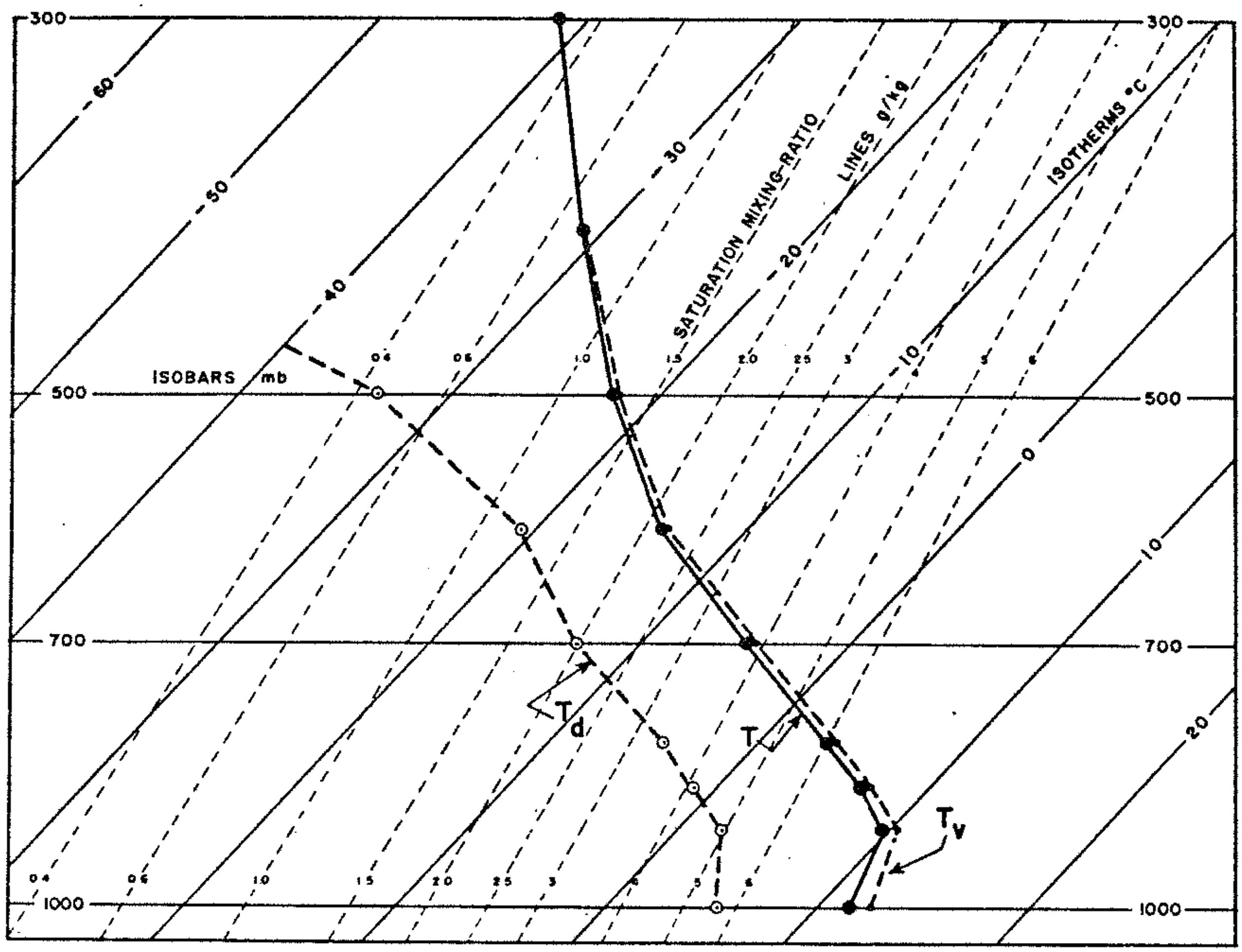

Figure 10. Comparison Between the Observed-Temperature and Virtual-Temperature Curves.

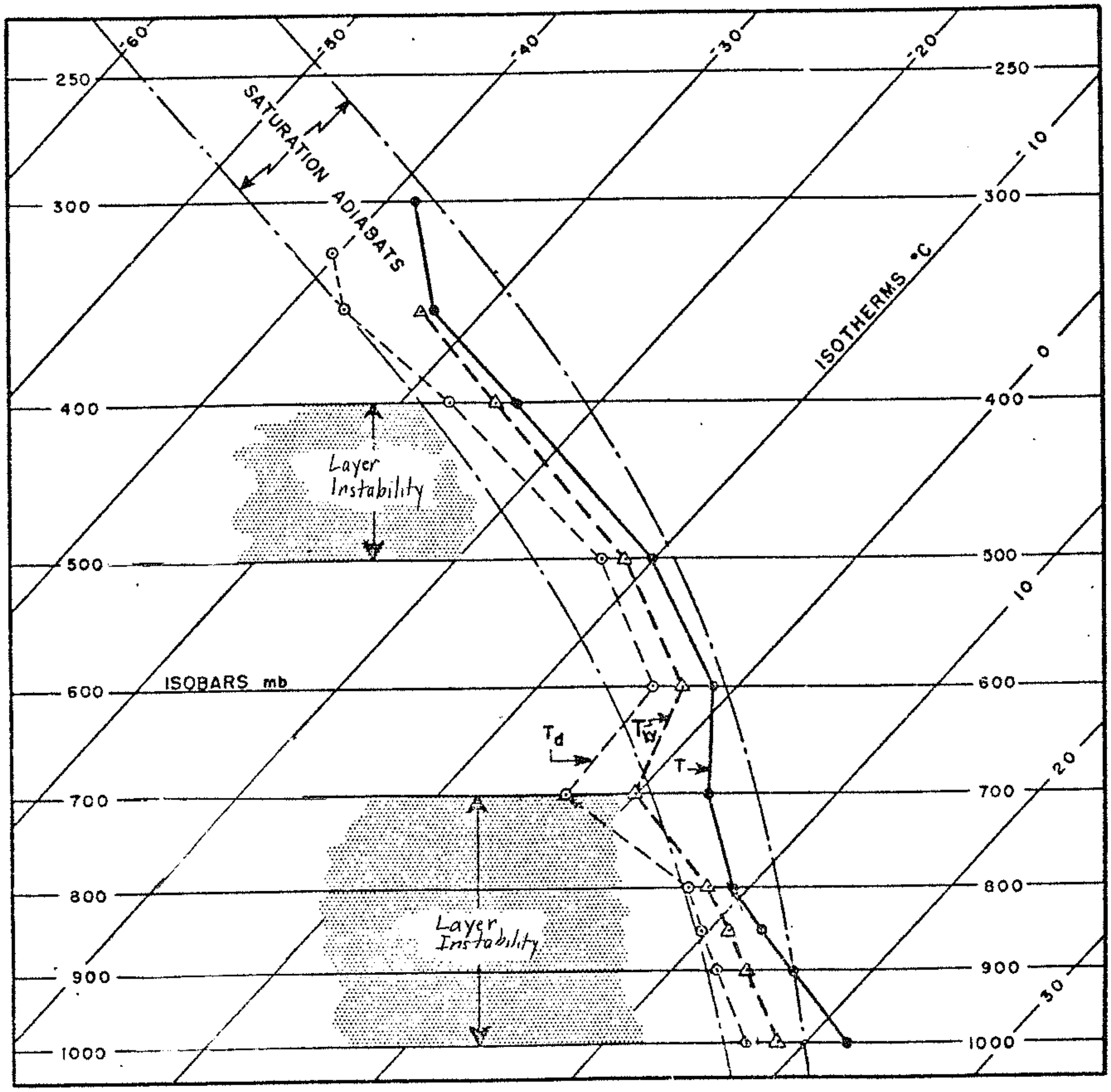

Figure 11 Example of Layer Instability, Comparing Use of T_w and T_d Curves for Diagnosis.

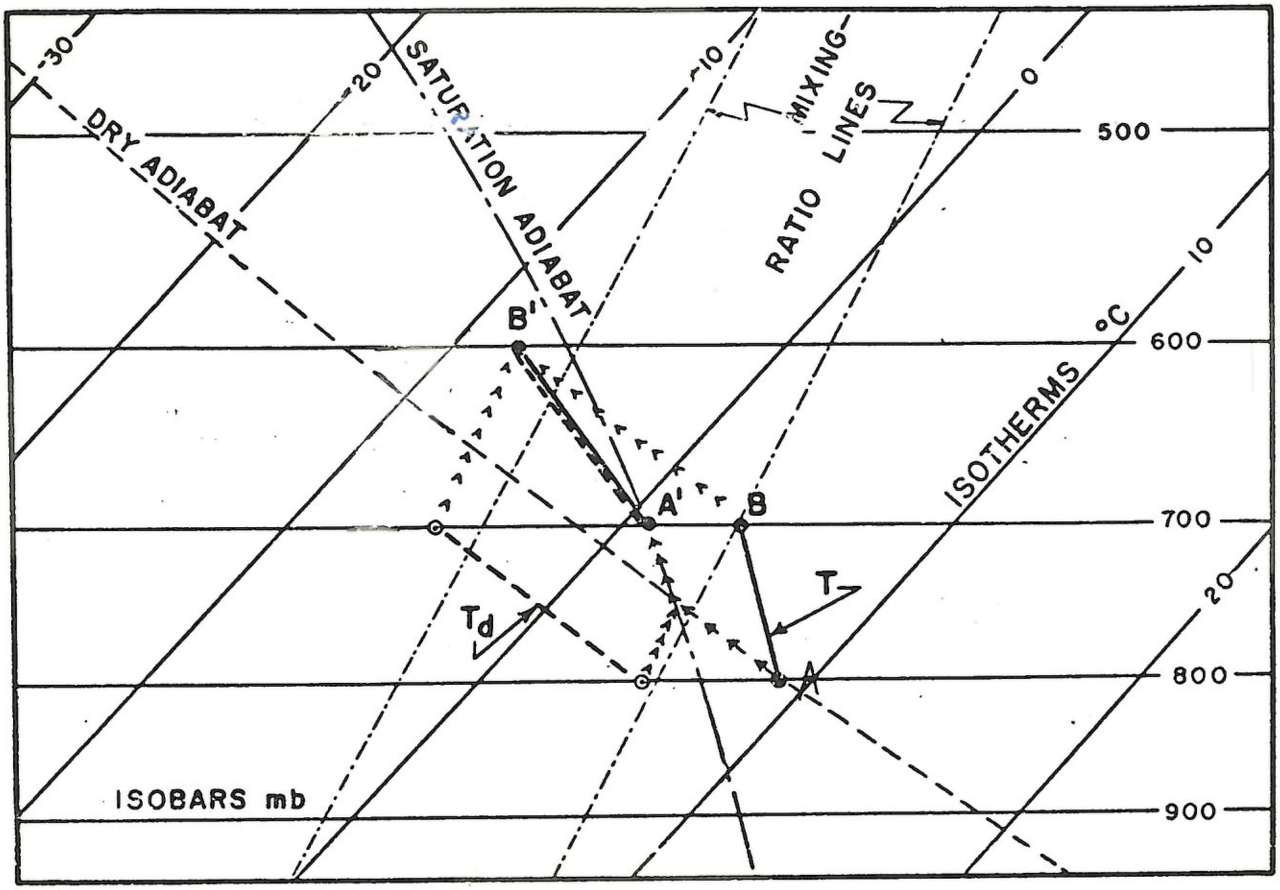

Figure 12. Example of a stable ~~layer~~ atmosphere in the parcel sense, but with layer instability.

Practically, this means that potentially unstable layers humid on the bottom part and dry on the top part. .ypical potentially unstable sounding superficially looks like a subsidence sounding described below. The presence of potential instability is determined with information on the vertical distribution of θ_w. A potentially unstable atmosphere becomes absolutely unstable when saturation occurs in the layer. Usually saturation is achieved by lifting of the layer, hence the designation "convective instability." However, saturation may be achieved also by evaporation of rain that falls through the layer.

Inversion

As described above, a layer where the temperature increases with height is an *inversion*. Inversions are particularly stable layers. A number of atmospheric phenomena are related to inversions. It is useful to distinguish several categories of inversions, as shown in Fig. 5-4. The main types of inversion are as follows.

> *Subsidence inversion*. Typically, the dew point decreases with height within the inversion. The humidity at the top of the inversion is low (Fig. 5-4a).
>
> *Frontal inversion*. In such an inversion the dew point increases with height. The air at the top of the inversion is humid (Fig. 5-4b).

Exceptions (of course) have been observed. The passive cold front often occurs under a subsiding warm air mass. Then the sounding looks like the subsidence model in Fig. 5-4a. Soundings in rainy weather may show "frontal inversion" even if the structure of the atmosphere shows that there are no fronts in the vicinity.

Other categories of inversions include the following.

> *Radiation inversion*. The temperature and dew point resemble the frontal inversion, except that the bottom of the inversion is at or very close to the ground. When a radiation inversion does not dissipate during the day, it may further intensify the next night. Strong and widespread radiation inversions of the polar night develop into the arctic air. The inversion on the top of the arctic air develops into the arctic front. That is the main mechanism of generation of arctic air.
>
> *Turbulent inversion*. This inversion forms on the top of an adiabatic layer. Turbulent eddies in the subjacent adiabatic layer cool the bottom of the turbulent inversion. The top of the turbulent inversion is not affected by turbulence and cooling. The subjacent adiabatic layer can be recognized by an adiabatic lapse rate and by a constant mixing ratio: The dew-point distribution follows the isopleths of constant saturation mixing ratio. An adiabatic inversion and its subjacent adiabatic layer are illustrated in Fig. 5-2 between 410 and 470 mb.

Again, as with all other models, we find many exceptions and intermediate cases when it is not easy to apply the rules. Some examples of interpretation of inversions and stable layers are shown in later sections.

5-6 INSTABILITY INDICES

For comparing stability in different places on the weather charts, it is convenient to represent instability (or stability) by a number. Such a number is usually called the *index of instability* (or *index of stability* for some indices). Other measures of instability are obtained by vertical integration of thermodynamic quan-

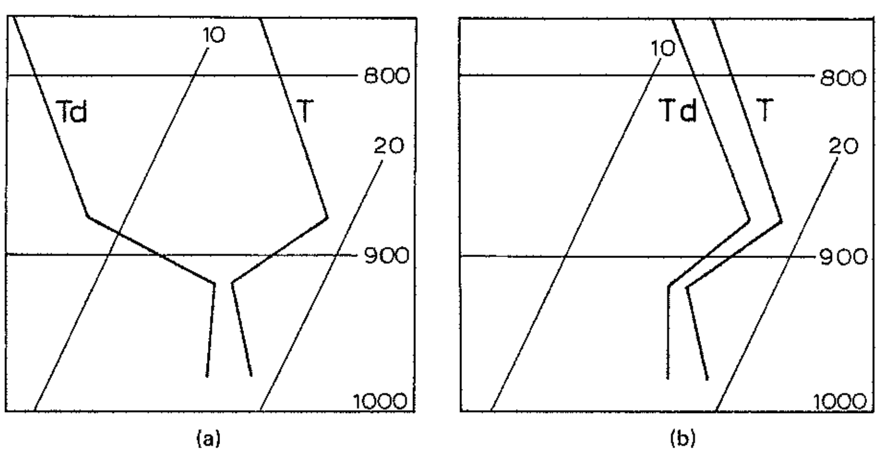

FIGURE 5-4. Vertical distribution of temperature and dew point in ses of (a) subsidence inver- ,nd (b) frontal inversion.

Appendix: Simplified expression for virtual temperature

First, a review of long division. What is x^2-3x-4 divided by $x+1$?

$$
\begin{array}{rl}
 & x \;\; -4 \\
x+1 \,\big) & x^2-3x-4 \\
 & -(x^2+x) \\
 & \quad -4x-4 \\
 & -(-4x-4) \\
 & \qquad 0
\end{array}
$$

Multiply by $x(x+1)$
Subtract
Multiply by $-4(x+1)$
Subtract

Check: $(x+1)(x-4) = x^2-4x+x-4 = \boxed{x^2-3x-4}$

Correct!

What is x^2+3x+5 divided by $x+1$?

$$
\begin{array}{rl}
 & x \;\; +2 \\
x+1 \,\big) & x^2+3x+5 \\
 & -(x^2+x) \\
 & \quad 2x+5 \\
 & -(2x+2) \\
 & \qquad 3
\end{array}
$$

3 ← Remainder

$$\boxed{\frac{x^2+3x+5}{x+1} = \underbrace{x+2}_{\text{quotient}} + \underbrace{\frac{3}{x+1}}_{\text{remainder}}}$$

Simplification of T_v by long division:

Let's assume mixing ratio q has units kg/kg so that T_v is now

$$T_v = \frac{1 + \frac{q}{0.622}}{1 + q} T \quad (1)$$

Multiply (1) by $\frac{0.622}{0.622}$ to simplify the expression as:

$$T_v = \frac{0.622 + q}{0.622(1+q)} T \quad (2)$$

Divide $0.622+q$ by $0.622(1+q)$

$$\begin{array}{rl} & 1 + 0.608q \\ 0.622(1+q) \,\big) & 0.622 + q \\ & -(0.622 + 0.622q) \\ \hline & 0.378q \\ & -(0.378q + .378q^2) \\ \hline & -.378q^2 \end{array}$$

Multiply by $.622 + .622q$

Subtract

Multiply by $\frac{0.378}{0.622} q = .608q$

Subtract

The remainder is:

$\frac{-.378q^2}{0.622(1+q)}$, but $q^2 \approx 10^{-6} \frac{kg}{kg}$ and can be neglected.

Therefore: $\boxed{T_v \approx (1 + 0.608q)T}$ (3)