Equivalent potential temperature overview

The equivalent potential temperature (θ_e) is the temperature a parcel of air would have if it all its moisture were condensed out by a most adiabatic process (with the latent heat release being used to heat the air parcel), and the parcel is then brought dry adiabatically back to a reference level (1000 mb when applied in meteorology).

The equivalent temperature (T_e) is the same process, except its brought dry adiabatically back to its original pressure level.

The procedure on a thermodynamic chart is to lift an air parcel to its LCL, lift to the top of the chart where the moist and dry adiabats parallel, then the air parcel is brought back to its original level (for T_e) or to 1000 mb (for θ_e).

SkewT exercise

Initial values: p=980 mb; T=27°C=300.15°K, T_D=11°C=284.15°K, q=8.4 g/kg. Assume T_{LCL}=280°K=7.5°C , p_{LCL}=775 mb.

Calculations

This is a value where calculations often yield better precision than a SkewT. The graphical part involves large movements up and down with altitude, and it's easy to lose accuracy. The biggest mistakes when calculating T_e or θ_e include:1) not putting temperatures in units Kelvins; 2) Not using the temperature at LCL; 3) Not using q at the LCL, which incidentally is the same as q at the original pressure level (it's common to mistakenly use q_s at the original pressure level). Also, I include a 1000 conversion so q can be used in unit g/kg. Other textbooks do not include this conversion.

The conventional equations are:

$$T_e = Texp\left[\frac{Lq}{1000c_pT_{LCL}}\right]$$

$$\theta = T\left(\frac{1000}{p}\right)^{R/c_p}$$

$$\theta_e = \theta exp\left[\frac{Lq}{1000c_pT_{LCL}}\right]$$

with latent heat of vaporization L=2.5 $\times$ 10^6 J kg^{-1}, the gas constant R=287 J kg^{-1} K^{-1},and specific heat at constant pressure c_p=1004 J kg^{-1} K^{-1}.

The standard θ_e underestimates values in the tropics. It's better to use Bolton's equivalent potential temperature equation:

$$\theta_e = \theta exp\left[q(1 + 0.81 \times 10^{-3}q)\left(\frac{3.376}{T_{LCL}} - 0.00254\right)\right]$$

Answers:

$$T_e = (300.15)exp\left[\frac{(2.5 \times 10^6)(8.4)}{(1000)(1004)(280)}\right]$$

$T_e = 323.48$°K=50.3°C

$$\theta = (300.15)\left(\frac{1000}{980}\right)^{287/1004}$$

$\theta = 301.88$°K=28.7°C

$$\theta_e = (301.88)exp\left[\frac{(2.5 \times 10^6)(8.4)}{(1000)(1004)(280)}\right]$$

$\theta_e = 325.36$°K=52.2°C

And for Bolton's formulation

$$\theta_e = (301.88)exp\left[8.4(1 + 0.81 \times 10^{-3}(8.4))\left(\frac{3.376}{280} - 0.00254\right)\right]$$

$\theta_e = 327.22$°K=54.1°C

A "back of envelope" rule of thumb states $\theta_e \approx 1.1\theta$. This should only be used for very rough numbers. Note the ratio for the conventional θ_e gives 1.077, and for Bolton's θ_e gives 1.084.